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Geometry of aggregates

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A SURVEY OF FURTHER DEVELOPMENTS

Linear systems consisting of ∞^3 and ∞^4 two-dimensional contravariant aggregates in S_3 have also been investigated by means of the theory developed in Chapters I, II and III. A brief survey of these investigations follows.

§ 1. The systems of aggregates $]3|2[_0 \cap S_3$.

The series $]3|2[_0 \cap S_3$ contains ∞^1 simply degenerated aggregates $]2[$. Then the series $]2|3[_0 \cap S_3$ resting on the former series contains ∞^1 simply degenerated $]3[$. The vertices of these stars $]3[$ form the manifold $\overline{]2|3[_0 \cap S_3$ which is a curve of order 6 and genus 3, ${}^3C_1^6$. The vertices of the degenerated $]2[$ of $]3|2[_0 \cap S_3$ are trisecants of this curve and generate a surface of order 8 containing ${}^3C_1^6$ counted three times.

The series $]2|3[_0 \cap S_3$ possesses ∞^2 subseries $]1|3[_0 \cap S_3$. A series $]1|3[_0 \cap S_3$ rests on a manifold $\overline{]3|1[_0 \cap S_3$ which gives a tetrahedral complex of lines. Such a complex contains as singularities four sheaves of lines and four planes of lines, the carriers of which are respectively the vertices and sides of a tetrahedron, inscribed in ${}^3C_1^6$. The surface of trisecants of ${}^3C_1^6$ turns out to be the intersection of ∞^2 tetrahedral complexes of lines. The planes of the ∞^1 simply degenerated $]2[$ of $]3|2[_0 \cap S_3$ envelop a surface of class 8. The planes containing two trisecants are the double planes of this prime-surface. Every point of ${}^3C_1^6$ belongs to 18 such planes. The above-mentioned prime-surface has a pencil of class 6 in common with every $]3[$ of $]2|3[_0 \cap S_3$. It turns out that this pencil is projectively equivalent to the curve $\overline{]2|3[_0 \cap S_3} = {}^3C_1^6$. The birational relation between the manifolds $\overline{]2|3[_0 \cap S_3}$ and $\overline{]3|3[_0 \cap S_2}$ represents ${}^3C_1^6$ on a plane curve ${}^3C_1^4$. We obtain a clear insight into this relation if we consider it as a part of the representation of the Bordiga surface $\overline{]2|3[_0 \cap S_4} = F_2^6$ by a plane.

Intersections of F_2^6 by primes give a set of ∞^4 curves ${}^3C_1^4$ in S_2 all passing through 10 base points.

Then the remaining intersection of two curves ${}^3C_1^4$ consisting of 6 points represents an intersection of ${}^3C_1^6$ by a plane. The image curve ${}^3C_1^4$ of ${}^3C_1^6$ is intersected in four points by a line; these points are the images of the four vertices of the tetrahedron of singularities belonging to a subseries $]1|3[_0 \cap S_3$ of $]2|3[_0 \cap S_3$. Moreover, one side of such a tetrahedron also contains a trisecant; thus by associating this trisecant with the fourth vertex of the tetrahedron we get a one-one correspondence between the points of ${}^3C_1^6$ and the trisecants of ${}^3C_1^6$. The birational relation between $]3|2[_0 \cap S_3$ and $]3|2[_0 \cap S_3^*$ affords an image of S_3 on S_3^* whereby the points of ${}^3C_1^6$ in S_3 are represented on the trisecants of ${}^3C_1^6$ in S_3^* and vice versa. A surface of order n in S_3 is represented by a surface of order $3n$ in S_3^* .

By means of a representation of $]3|2[_0 \cap S_3$ in the matrix-space S_{11} we see that ${}^3C_1^6$ in S_3 is the projection of a curve of order 10 in S_{11} .

§ 2. The system of aggregates $]4|2[_0 \cap S_3$.

Resting on the series $]4|2[_0 \cap S_3$ we have a series $]2|4[_0 \cap S_3$. All aggregates of the latter are degenerated and 10 aggregates are two-ply degenerated; the vertices form the manifold $]2|4[_0 \cap S_3$ consisting of 10 points K_1, \dots, K_{10} .

Thus $]4|2[_0 \cap S_3$ also contains ∞^2 simply degenerated $]2|$, the vertices forming a congruence of lines $|d|$ of order 3 and class 6. The 10 points K_p are the vertices of cones of order 3 of the congruence $|d|$. Paired with every two-ply degenerated $]4|$ is a regulus of lines which are the vertices of degenerated $]2|$. Every one of these reguli defines a quadratic surface containing 9 of the 10 points K_p .

We can immediately determine the mutual positions of manifolds which are connected with subseries of $]4|2[_0 \cap S_3$. The system $]2|4[_0 \cap S_3$ contains ∞^2 subseries $]1|4[_0 \cap S_3$. The manifold $]4|1[_0 \cap S_3$ connected with a similar subseries consists of the entire set of lines in S_3 as well as a pencil of planes of class 3, the planes representing the degenerated $]1|$ of the series $]4|1[_0 \cap S_3$ which rests on $]1|4[_0 \cap S_3$. If we take a fixed $]4|$ of $]2|4[_0 \cap S_3$, then it belongs to ∞^1 subseries $]1|4[_0 \cap S_3$. The ∞^1 pencils of planes of class 3 connected with these subseries envelop a surface of class 5.

The birational relation between the manifolds $\overline{]4[2[}_0 \cap S_3$ and $\overline{]3[2[}_0 \cap S_4$ induces a one-one correspondence between the lines of the congruence $|d|$ and the points of the Bordiga surface F_2^6 in S_4 . At the same time there is a one-one correspondence between $\overline{]2[3[}_0 \cap S_4 = F_2^6$ and $\overline{]4[3[}_0 \cap S_2 = S_2$ already mentioned in the preceding paragraph. Many properties of F_2^6 as deduced in Room (1) Chapter XIV, can now be proved by means of the above-mentioned correspondences. Conversely the Bordiga surface gives a clear representation of the congruence $|d|$ in S_3 .

The investigation of the linear systems of two-dimensional contravariant aggregates will be complete if all systems $\overline{]n[2[}_0 \cap S_3$ ($n = 1, \dots, 10$) have been investigated.

The theory regarding complementary systems of aggregates is of great importance in this investigation. In this case the systems are $\overline{]n[2[}_0 \cap S_3$ and $\overline{]10-n[2[}_0 \cap S_3$. For instance let us take $n = 8$ as an example. The system $\overline{]8[2[}_0 \cap S_3$ contains a set of ∞^2 completely degenerated stars $\overline{]2[}$. This set of planes is dual to the point-manifold $\overline{]2[2[}_0 \cap S_3$. The planes envelop a surface of class 3.